Exercise 6.5 – Computation of homopolar impedance of ungrounded MV networks

For the electrical distribution network represented in the figure below, you must:

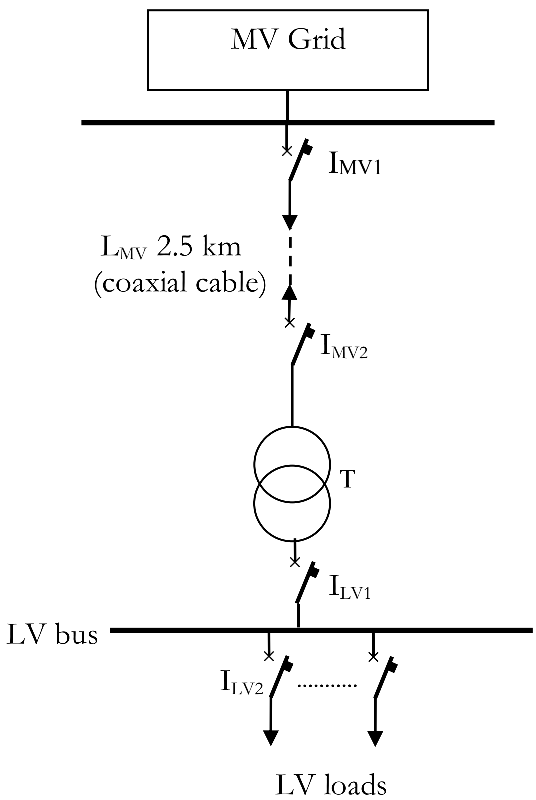
1. define and verify the characteristics of the medium-voltage (MV) cable L_{MV} (EPR insulation), considering a maximum intervention time of $t_{sc}^{I_{MV1}} = 300ms$ for circuit breaker I_{MV1} after a three-phase short circuit.
2. Determine the direct, inverse and homopolar impedances of the medium-voltage network without considering the presence of the cable L_{MV}.
3. Determine the homopolar impedance of the medium-voltage network considering the presence of the cable L_{MV} and re-compute the phase-to-ground short circuit current of the MV grid.

Medium-voltage grid data (MV grid):

- Nominal voltage: $V_n^{MV} = 20\ kV$
- Neutral: ungrounded
- Short-circuit power: $S_{sc}^{MV} = 500\ MVA$
- Phase-to-ground short circuit current of the MV grid: $I_{sc,ph-gnd}^{MV} = 75\ A$
- Ratio $R_{sc}^{MV}/X_{sc}^{MV} = 0$

MV/LV transformer T data:

- Nominal power: $S_n^T = 400\ kVA$
- Nominal transformer ratio: $V_{n1}^T/V_{n2}^T = 20/0.4\ kV$
- Winding connection: Delta (20 kV winding) – Star grounded (0.4 kV winding)
- Short circuit voltage $V_{sc}^T = 0{,}05pu$
- Short circuit copper losses $P_{sc}^T = 4.5\ kW$



Characteristics of medium voltage cables (20kV) with EPR insulation (Joule integral $K = 143\ \frac{A \cdot s^{\frac{1}{2}}}{mm^2}$).

Cross section [mm²]	Resistance pul [Ω/km]	Reactance pul [Ω/km]	Shunt capacitance pul [μF/km]	Maximum Current [A]
25	0.929	0.15	0.18	157
35	0.670	0.14	0.17	190
50	0.495	0.13	0.19	228
70	0.344	0.13	0.21	284
95	0.248	0.12	0.23	346
120	0.198	0.12	0.25	399

Solution

Q1 – In order to define the characteristics of the MV cable connecting the MV grid with the transformer, we need to first select the cross section of the MV cable to withstand the rated current of the transformer on its MV side.

$$I_n^T = \frac{S_n^T}{\sqrt{3}V_{n1}^T} = \frac{400 \cdot 10^3 VA}{\sqrt{3} \cdot 20 \cdot 10^3 V} = 11.54\ A$$

By making reference to the provided characteristics of medium voltage cables (20kV) with EPR insulation, the cable with the minimum cross section$A_{L_{MV}} = 25\ mm^2$ is sufficient to supply the transformer at its nominal current since this cable has a maximum current of 157 A. However, we need to **verify whether this cable can withstand the Joule integral inequality associated to a three-phase short circuit at the beginning of the cable** and considering a maximum intervention time of $t_{sc}^{I_{MV1}} = 300ms$ for the circuit breaker I_{MV1}, namely:

$$\left(I_{sc,3ph}^{MV}\right)^2 t_{sc}^{I_{MV1}} \leq \left(K_{L_{MV}}\right)^2 \left(A_{L_{MV}}\right)^2$$

Let us first compute the three-phase short circuit at the beginning of the cable. For this computation, we can directly use the short circuit power of the MV supplying grid (indeed, there are no other lines between the beginning of the cable and the supplying grid).

$$I_{sc,3ph}^{MV} = \frac{S_{sc}^{MT}}{\sqrt{3}V_n^{MV}} = \frac{500 \cdot 10^6 VA}{\sqrt{3} \cdot 20 \cdot 10^3 V} = 14.4\ kA$$

Therefore, we can compute the minimum cross section of the cable satisfying the Joule integral inequality.

$$A_{L_{MV}} \geq \sqrt{\frac{\left(I_{sc,3ph}^{MV}\right)^2 t_{sc}^{I_{MV1}}}{\left(K_{L_{MV}}\right)^2}} = \sqrt{\frac{(14.4 \cdot 10^3 A)^2 \cdot 0.3s}{\left(143 \frac{A \cdot s^{\frac{1}{2}}}{mm^2}\right)^2}} = 55.2\ mm^2$$

By looking at the table with the characteristics of medium voltage cables (20kV) with EPR insulation, the first one that satisfies the above inequality has a cross section of 70 mm^2 and has the following parameters:

- Resistance pul $r_{pul} = 0.344\ \frac{\Omega}{km}$
- Reactance pul $x_{pul} = 0.13\ \frac{\Omega}{km}$
- Shunt capacitance pul $c_{pul} = 0.21\ \frac{\mu F}{km}$
- Shunt susceptance $X_{pul}^{sh} = -j\frac{1}{2\pi f \cdot c_{pul}} = -j\frac{1}{2\pi f \cdot 0.21 \cdot 10^{-6} F} = -j15.16\ \mathrm{k}\Omega \cdot km$

And for the whole cable

- Total cable resistance $r_{L_{MV}} = 0.344\ \frac{\Omega}{km} \cdot 2.5\ km = 0.86\ \Omega$
- Total cable reactance $x_{L_{MV}} = 0.13\ \frac{\Omega}{km} \cdot 2.5\ km = 0.325\ \Omega$
- Total cable shunt capacitance $c_{L_{MV}} = 0.21\ \frac{\mu F}{km} \cdot 2.5\ km = 0.525\ \mu F$
- Total cable shunt susceptance $X_{L_{MV}}^{sh} = -j\frac{1}{2\pi f \cdot c_{L_{MV}}} = -j\frac{1}{2\pi f \cdot 0.525 \cdot 10^{-6} F} = -j6.06\ \mathrm{k}\Omega$

Q2 – From the definition of short circuit power of a network $\left(S_{sc}^{MV} = \sqrt{3} V_n^{MV} I_{sc,3ph}^{MV}\right)$and its corresponding three phase short circuit current $I_{sc,3ph}^{MV}$, we can compute the short circuit impedance of the MV grid at the direct and inverse sequence.

$$|\bar{Z}_{sc}^{MV}| = |\bar{Z}_d^{MV}| = |\bar{Z}_i^{MV}| = \frac{V_n^{MV}}{\sqrt{3} I_{sc,3ph}^{MV}} = \frac{20 \cdot 10^3 V}{\sqrt{3} \cdot 514.4 \cdot 10^3 A} = 0.8\ \Omega$$

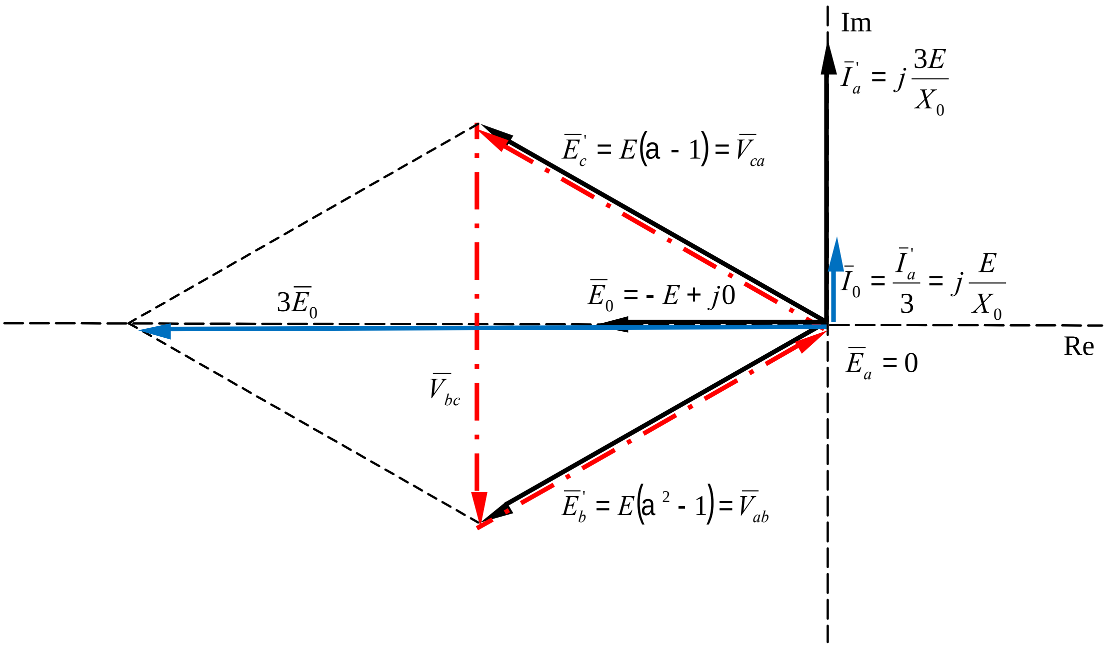
Furthermore, since $R_{sc}^{MV}/X_{sc}^{MV} = 0$, the phasor $\bar{Z}_{sc}^{MV}$ is simply:

$$\bar{Z}_{sc}^{MV} = R_{sc}^{MV} + jX_{sc}^{MV} = j0.8\ \Omega$$

For the homopolar sequence, the text of the exercise provides the phase-to-ground short circuit current of the MV grid $I_{sc,ph-gnd}^{MV} = 75\ A$. Now, from the theory on short circuit calculus, we know the following relationship linking the phasor of the phase-to-ground short circuit current, the network sequence impedances and the pre-fault phasor of the voltage where the fault occurs:

$$\bar{I}_{sc,ph-gnd}^{MV} = \frac{3\bar{E}}{\bar{Z}_d^{MV} + \bar{Z}_i^{MV} + \bar{Z}_0^{MV}}$$

Now, considering the pre-fault phasor of the voltage where the fault occurs as the reference one (i.e., we place $\bar{E}$ on the real axis), we do know from the theory the location of the phasors of the post-fault phase-to-ground voltages, phase-to-phase voltages, homopolar voltage and current as well as the fault current (see figure below taken from the lecture 6.6).



Therefore, we can write

$$\bar{I}_{sc,ph-gnd}^{MV} = \frac{3\bar{E}}{\bar{Z}_d^{MV} + \bar{Z}_i^{MV} + \bar{Z}_0^{MV}} = j75A$$

And

$$\bar{Z}_0^{MV} = \frac{3\bar{E}}{\bar{I}_{sc,ph-gnd}^{MV}} - (\bar{Z}_d^{MV} + \bar{Z}_i^{MV}) = \frac{3\frac{20\cdot 10^3 V}{\sqrt{3}}}{j75A} - 2(j0.8\ \Omega) = -j463.5\ \Omega$$

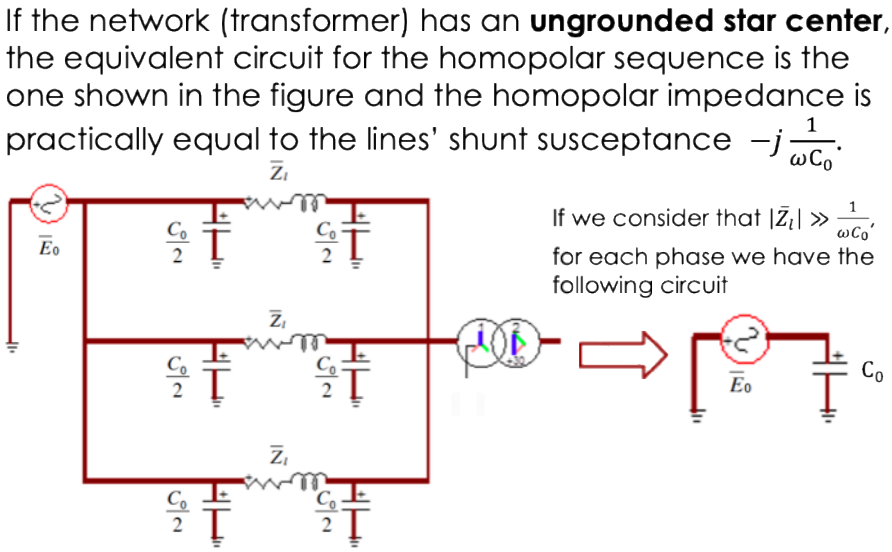
Alternatively, we can compute the $\bar{Z}_0^{MV}$ by considering that $|\bar{Z}_0^{MV}| \gg |\bar{Z}_d^{MV}|, |\bar{Z}_i^{MV}|$ and the fact that, for ungrounded networks, we know that $\bar{Z}_0^{MV} \approx -jX_0^{MV}$. Therefore, we can write the following approximated computation:

$$\bar{I}_{sc,ph-gnd}^{MV} = \frac{3\bar{E}}{\bar{Z}_d^{MV} + \bar{Z}_i^{MV} + \bar{Z}_0^{MV}} \approx \frac{3\bar{E}}{jX_0^{MV}}$$

Therefore, we have

$$jX_0^{MV} \approx \frac{3\bar{E}}{\bar{I}_{sc,ph-gnd}^{MV}} = \frac{3\bar{E}}{\bar{I}_{sc,ph-gnd}^{MV}} = \frac{3\frac{20\cdot 10^3 V}{\sqrt{3}}}{j75A} = -j461.9\ \Omega$$

Q3 – To determine the homopolar impedance of the medium-voltage network considering the presence of the cable L_{MV}, we must observe that the inclusion in the MV grid of the 2.5 km cable produces a decrease of the grid homopolar sequence impedance. Indeed, since the grid has an *ungrounded neutral*, its homopolar sequence impedance is composed by the shunt susceptances of the lines/cables (see figure lecture from lecture 6.5).



Therefore, cable shunt susceptance $X_{L_{MV}}^{sh}$ is in parallel with the grid homopolar impedance $\bar{Z}_0^{MV}$. Therefore, we can compute the new value of the **(grid + L$_{MV}$) homopolar impedance** as follows (ee remind that the total cable shunt susceptance $X_{L_{MV}}^{sh} = -j6.06$ kΩ):

$$\bar{Z}_0^{MV+L_{MV}} = \frac{\bar{Z}_0^{MV} \cdot X_{L_{MV}}^{sh}}{\bar{Z}_0^{MV} + X_{L_{MV}}^{sh}} = \frac{(-j461.9\ \Omega)\cdot(-j6060\ \Omega)}{(-j461.9\ \Omega) + (-j6060\ \Omega)} = -j430.6\ \Omega$$

As expected, the presence of the cable **reduces the magnitude of $\bar{Z}_0^{MV+L_{MV}}$**.

The new phase-to-ground short circuit current of the MV grid taking into account the presence of the cable is therefore:

$$\bar{I}_{sc,ph-gnd}^{MV} = \frac{3\bar{E}}{\bar{Z}_d^{MV} + \bar{Z}_i^{MV} + \bar{Z}_0^{MV+L_{MV}}} = \frac{3\,\frac{20\cdot 10^3 V}{\sqrt{3}}}{j0.8\ \Omega + j0.8\ \Omega - j430.6\ \Omega} = j80.8\ A$$

. Or, in a more approximated way,

$$\bar{I}_{sc,ph-gnd}^{MV} \approx \frac{3\bar{E}}{\bar{Z}_0^{MV+L_{MV}}} = \frac{3\,\frac{20\cdot 10^3 V}{\sqrt{3}}}{-j430.6\ \Omega} = j80.4\ A$$

As expected, the presence of the cable **increases the magnitude of the phase-to-ground short circuit current of the MV grid**.